Techniques of Scale Construction

Rabindra Nath Padaria
Sujit Sarkar
V.Lenin

Division of Agricultural Extension
ICAR-Indian Agricultural Research Institute,
New Delhi -110012

Techniques of Scale Construction

Rabindra Nath Padaria
Sujit Sarkar
V.Lenin

Division of Agricultural Extension
ICAR-Indian Agricultural Research Institute,
New Delhi -110012

Composed by:
M. S. Printers, C-108/1 Back Side, Naraina Industrial Area, Phase-1, New Delhi-110024
Mob: 7838075335, 9990785533, 9899355565, Tel.: 011-45104606
E-mail: msprinter1991@gmail.com

Content

Chapter 1

CONCEPTS AND TECHNIQUES OF SCALE CONSTRUCTION

Measurement is most commonly understood as the assignment of numerals to objects or events according to rules as per the definition of Stevens. The process is straight while dealing with physical sciences, where there are standardized and calibrated mechanism to measure the traits and parameters and the task is comparatively easy and more precise. However, dealing with the psychological and social conceptions, where measurement rests upon qualitative observations, the researchers have to specify the dimension, develop constructs and their operational definitions and develop suitable tools for their measurement. The selected variables and standardized tools are tested for internal and external validity for generalization. Despite these efforts, the tools and results may not get acceptance among the peer researchers. Such limitations always motivate social scientists for elaborate groundwork for developing and testing their tools and techniques to ensure acceptability among fellow researchers and other end users. Measurements in terms of time, distance, or energy reflect the observation of physical properties of both the stimulus and response. However, when measurement is to be done for psychological variables, we find lack of appropriate tools. Psychological scaling has been widely researched and used for measurement of psychological attributes. The chief purpose of psychological scaling methods has been to evaluate stimulus objects on linear scales (Guilford, 1987). A scaling study is the process of establishing scale values. Scale is like a measuring instrument. A scale is a continuum consisting of the highest point and the lowest point in terms of a characteristic i.e. favourableness, agreement, etc. (Wilkinson and Bhandarkar, 1977). Kerlinger (1983) defines scale as a set of symbols or numerals so constructed that the symbol of numerals can be assigned by rule to the individuals to whom the scale is applied. In extension research, attitude scales are popularly used as very often necessity is there to make distinction of degree and researchers need to ascertain whether one individual is more favourable to an issue than the other.

Attitude scale and its construction

Attitudes are literally mental postures, a guide for conduct to which each new experience is referred before a response is made (Morgan, 1934). Attitude is defined as the degree of positive or negative affect associated with some psychological object. (Edwards, 1969)). Pioneering works of Thurstone, Likert and Guttman have a great bearing upon the techniques of attitude scale construction. The attitude scales differ from other tests in the sense that in latter a person either passes an item or fails, while in case of attitude scale a respondent accepts or endorses opinion with respect to the item at some region on the continuum.

To make an assessment of attitude of people one can ask **direct questions**. But direct questioning sometime results in untrue response e.g., if a person is asked: do you like your teacher? His reply in the presence of the teacher of his colleagues may be" 'I like my teacher very much'. He may not reveal his true feeling out of fear of losing marks, or to please the friends or his teacher or just to remain out of any controversy. Another way of assessing the attitude of an individual is by **observing his behavior.** But it is possible that the individual may not be behaving in accordance with his attitude. He may dislike his teacher but, in the presence of his teacher he praise him and shows good manners but as soon as the teacher is gone he may start abusing the teacher. In this case both kinds of behaviors are demonstrated. If one observes the individual in the presence of the teacher one may draw a different conclusion as the one draw soon afterwards. Thus observation of behavior may also be misleading. Besides it is not possible to observe large number of individuals under specific conditions. There is therefore no necessary one to one correspondence between overt behavior and attitudes. Besides, direct questioning and observation at best can help us classify people into two or three categories i.e., having favourable attitude, unfavorable attitude and in- between the two categories. For the purpose of research, if one is interested in relating one variable to another variable there is need for greater degree of refinement in the system of classification. Hence, for a quick and convenient measure of attitudes that can be used with large groups has led to the development of attitude scales. These scales provide us with a means of obtaining as assessment of the degree of affect that individuals may associate with psychological object.

An attitude scale consists of a number of items/that have been carefully selected and edited in accordance with certain criteria. The items making an attitude scale are called statements. A statement is anything that is said about a psychological object. The class of all possible statements that could be made about a given psychological object is often called a universe of content or simply a universe. One of the major assumptions involved in the construction of an attitude scales is that there is a difference in the belief and disbelief system of individuals with favorable attitude towards some psychological object and those with unfavorable attitude.

Steps in scale construction

Generally the following steps are followed in the process of construction of a scale:

1. Item collection
2. Item selection
3. Ordering of the items on a psychological continuum
4. Testing the reliability and validity of the constructed (ordered) scale.
5. Administration of the scale

Item collection- To collect the items for the scale, first of all a clear understanding of the universe of content pertaining to the psychological object must be arrived at, e.g., if a scale is being constructed on attitude towards High yielding Variety (HYV), then all aspects of attitude to HYV in terms of its quality, productivity, input requirements, usage as food, fodder, color, texture, etc must first be delineated and then items pertaining to all the aspects collected.

The selection of statement as items of the scale should be such that there is difference in endorsement of the statements by those who have unfavorable attitude vis-à-vis those having favorable attitude. The statement therefore should be nonfactual, but expressing favorable/unfavorable feeling about the psychological object for which it is constructed.

Own thought, review of published materials- including popular articles, newspaper articles, research papers, books, etc., consultation with experts, colleagues, respondents, etc and sometimes conducting a pilot study may be considered for identifying and collecting the items (statements). After collecting the items, these should be further scrutinized and refined. The items collected for inclusion in the scale should be edited according to the 14 criteria of Edwards and Kilpatrick (1948) as quoted by Edwards (1969). They are given as below:

1. Avoid statements that refer to the past rather than to the present.
2. Avoid statements that are factual or capable of being interpreted as factual.

3. Avoid statements that may be interpreted in more than one way.
4. Avoid statements that are irrelevant to the psychological object under consideration.
5. Avoid statements that are likely to be endorsed by almost everyone or by almost no one.
6. Select statements that are believed to cover the entire range of the effective scale of interest.
7. Keep the language of the statements simple, clear and direct.
8. Statements should be short, rarely exceeding 20 words.
9. Each statement should contain only one complete thought.
10. Statement containing universals such as all, always, none and never often introduce ambiguity and should be avoided.
11. Words such as only, just, merely and others of a similar nature should be used with care and moderation in writing statements.
12. Whenever possible, statements should be in the form of simple senetences rather than in the form of compound or complex sentences.
13. Avoid the use of words that may not be understood by those who are to be given the completed scale.
14. Avoid the use of double negatives.

Once the final list of items/statements is decided upon, these have to be ordered on a psychological continuum. There are several methods that one can follow to order the items on the psychological continuum based on judgment of experts. It requires understanding of various scale construction techniques. However, in extension research the various methods generally used for construction of attitude scale includes: a) method of paired comparison, b) equal appearing methodology, c) successive interval methodology and d) method of summated rating.

(A) Paired Comparison Method

Thurstone made significant contribution in scale construction with his law of comparative judgment. Paired comparison and equal interval methods draw heavily on assumptions of law of comparative judgment. In this method statements/items selected for the scale are given to the judge in pairs. Each and every statement is paired with all others. e.g., if the number of statements is 7 then we shall have 7*6/2 pairs of statements i.e., 21 pairs. Optimum number of statements to be included in a scale being constructed by the paired comparison methodology ranges from 7 to 10. Let n be the number of statements then the number of pairs will be n*(n-1)/2. As the statements increase the number of pairs also increases manifold.

If we give these n (n-1)/2 pairs of statements of 30 to60 individuals and ask them to make comparative judgment as to which statement in each pair is more favorable, the data so collected will provide us with frequencies corresponding to the number of times that each statement or item is judge more favorable than every other statement. The data will be in the form as given below for N=100 judges/experts who are asked to provide the judgment say for 4 statement i.e. 6 pairs and in each pair to mark the statement that reflects more favorableness towards the psychological object. After collecting judgment of each judge the data is compiled in the form of frequencies. Let us consider four statements (A, B,C and D) and the judgment of paired statements. Suppose with respect to A-B pair of statement, 40 judges rated A to be more favorable than B, while 60 judges rated B to be more favorable than A., the data will be presented as Table-1 below. Similarly the data for every pairs of statements will be obtained and tabulated schematically into a matrix as represented below:

Table-1: Frequency (F_{ij}) matrix

F_{ij}

i

j		A	B	C	D
	A	50	60	75	90
	B	40	50	70	95
	C	25	30	50	65
	D	10	5	35	50

(I judged>j) N=100; N= number of experts of express/judges)

It should be noted that entries in f_{ij} cells 1-1, 2-2, 3-3 and 4-4 are filled by placing 50% of N to complete the matrix. After obtaining the frequency table we develop proportion matrix table P_{ij} (Table-2) by dividing the frequency entered in each cell by total number of judges, i.e.100.

Table-2: Proportion (P_{ij}) matrix)

$\mathbf{P_{ij}}$

i

j		A	B	C	D
	A	.50	.60	.75	.90
	B	.40	.50	.70	.95
	C	.25	.30	.50	.65
	D	.10	.05	.35	.50

After making the P_{ij} matrix the column entries are added and checked whether the sums of each column at the base of the matrix are in ascending order or not i.e, sum of column 1 should be the least and column 4 the last column should be the maximum. If not the matrix is rearranged to get ascending order of the sums of the column entries. From the proportion matrix we make a matrix of normal deviates. Each cell entry of proportions is replaced with corresponding normal deviates (Table-3). The conversion table (Edwards, 1957) may be referred.

Table-3: Normal deviates (Z_{ij}) corresponding to P_{ij}

Z_{ij}

	A	B	C	D
A	00	.253	.674	1.282
B	-.253	00	.524	1.645
C	-.674	-.524	00	.385
D	-1.282	-1.645	-.385	00
Sum	-2.209	-1.916	.813	3.312
Mean	-.552	-.479	.203	.828
Mean+ (0.552)	0	.073	.735	1.38

Summing up of the column entries and dividing by n (here 4) we get mean value for each column entry. This gives us the value of statements in terms of its distance from the mean of the continuum. The least value, i.e, the highest negative value is the farthest from the average of the continuum. When this value with positive sign is added to all other values we get the first value as zero and the scale value of other statement are obtained accordingly.

The method so far described only tells how to arrive at scale value for each statement. However, there are several checks to be applied to ensure that the scale values are valid.

Internal consistency check

Prior to using the scale, it is important to apply the internal consistency check. The internal consistency check involves determining whether or not the observed and empirical proportion P_{ij} expected from the derived scale value are in agreement. To carry out internal consistency checks a matrix of theoretical normal deviates corresponding to the scale separation of the statements are obtained.

A table is set up where the rows and columns are bounded by the scale values and the cells in the table are filled up by subtracting the entry on the left of the table (i.e the row) from the entry on the top of the table (i.e. the column) as shown in the Table-4 below.

Table-4: Theoretical Normal Deviate (Zij')

Zij'	.000	.073	.735	1.38
.000	000	-		
.073	.-.073	00		
.735	-.735	-.639	00	
1.38	-1.38	-1.312	-.673	00

Based on the theoretical normal deviation we obtain the corresponding theoretical (P_{ij}') proportions as given below (Table-5) and calculate the absolute average discrepancy between the observed proportions and theoretical proportions (P_{ij} - P_{ij}') as in Table-6.

Table-5: Theoretical (P_{ij}') proportions

P_{ij}'

A	B	C	D
-			
.418	-		
.199	.261	-	
.064	.095	.250	-

Table-6: Discrepancy between the observed proportions and theoretical proportions (P_{ij} - P_{ij}')

P_{ij} - P_{ij}'				
Statements	1	2	3	4
A	-			
B	0.082	-		
C	0.201	0.039	-	
D	0.186	-0.045	0.1	-
Σ	0.469	-0.006	0.1	-

Absolute Average Discrepancy (AD) is worked by following formula:

$$AD = \frac{.\Sigma\left|pij - pij'\right|}{\frac{n(n-1)}{2.}}$$

where n=4

2

$\Sigma[p_{ij}-p_{ij'}] = .575$

$AD = \frac{.575}{6} = .096$

The AD value equal to or less than .03 indicates a high degree of internal consistency. A test called chi square test of significance needs is applied to see whether the assumptions originally made regarding equality of standard deviations are tenable or not.

Test of significance for the case V model

Whether the assumptions of case V that the observed and theoretical proportions are in tune with each other or not is judged by ch-square($x^{2)}$ test using the formula as below:

$$\chi^2 = \frac{\sum(\theta - \theta')^2}{821/N}$$

N is number of judges.

The degree of freedom is computed by the formula : df = (n-1)(n-2)/2, where n is the number of statement.

Firstly the corresponding values of P_{ij} and P_{ij}' are converted through angular transformation table into θ and θ, respectively.

How to use the scale: The scaled statements are presented in random order to the respondents/subjects under study and are asked to select the statement with which a respondent agrees the most. The scale value of the statement chosen by the respondent is taken as his attitude score on the given psychological object.

Alternately a subject/respondent may be asked to choose three statements he agrees with. His score then can be taken as median of the three scale values of these selected statements. If a respondent is allowed to choose as many statements as he/she wants, then median value will be considered as his/her score in case of odd number of selected statements. If even numbers of statements are selected, then the mean of the scale values of middle two statements will be taken as his/her score. Another possible way of scoring an individual is to present the statements pairs e.g, each statement paired with every other and then in each pair he chooses one statement with which he agrees more. The number of times he has chosen a statement with higher scale value can be taken as his score, e.g., in case the scale comprises 7 statements then number of pairs would be 21. Subject may choose all 21 times a statement of higher value or at the other end all 21 times statement of lower value. Thus the range of scores would vary from

0-21 depending upon the number of times a statement of higher value is chosen by the subject in these 21 pairs.

(B) Equal Appearing Interval Method

In this method, unlike paired comparison, only one judgment for each statement is required. Thus this method takes less time and allows us to take a large number of statements, which can be scaled on a psychological continuum.

The selected and edited statements are placed on cards and presented to the judges for sorting on a 9 or 11 points continuum marked A to I or A to K. The extremes of the continuum and the center points are explained as representing most unfavorable, most favorable and neutral respectively. The distance between all intervals is assumed to be equal and assigned value of 1. The judges sort the statements according to the content of each statement and not according to his attitude or belief. The frequency table for each statement is made after the judges (say 100 judges) have given their judgment as presented in Table-7 . The frequencies are converted into proportions by dividing each frequency by total number of judges (N). Then cumulative proportions are worked out.

Table-7: Sorting of statement by judges (N=100)

	Sorting categories										
	A	B	C	D	E	F	G	H	I	J	K
	1	2	3	4	5	6	7	8	9	10	11
Statement 1											
f	2	4	10	8	12	26	14	10	8	4	2
p	.02	.04	.10	.08	.12	.26	.14	.10	.08	.04	.02
cp	.02	.06	.16	.24	.36	.62	.76	.86	.94	.98	1.00

f: frequency, p: proportion, cp: cumulative proportion

Scale values of the statements

The scale values of the statements may be taken as the medians of the corresponding cumulative proportion distribution of the continuum.

The median may be computed by formula:

$$S_i = l + \left(\frac{50 - \Sigma pb}{pw} \right) x\, \overline{w}.j.$$

S_i = Scale value of the i_{th} stimulus (statement)

L = Lower limit of the interval on the psychological continuum in which the median falls.

Σ pb = the sum of the proportions below the interval in which the median falls.

Pw = the proportion within the interval in which the medium falls

$\overline{w}.j.$ = the width of the interval on the psychological continuum

In addition to the scale value it is essential to know the extent of spread of responses of the judges on the given continuum. If the statement is judged by some to lie in interval 1 and by some in interval 11and almost equal numbers spread in between, obviously then the statement has been judged differently by different judge, hence there is lack of agreement amongst the judges in judging the position of statement on the continuum. This reflects some inadequacy in the statement in terms of giving different meaning to different people. Such statements are not worthy of inclusion in a psychological scale. It is therefore suggested that inter quartile range referred to as Q value ($Q = C._{75}-C._{25}$) should also be found out along with the scale value and only those statements should be selected, which have small Q values. However, it is also essential to ensure that the scale comprises statements representing all the intervals and if possible half of all intervals and should be approximately equidistant. Inter quartile range is calculated as $Q = C._{75}-C._{25.}$ The statement with high Q value is not considered for inclusion in the scale. The final scale may comprise 20 to 30 statements representing scale values from each of the 11 points of the continuum. The scale is used the same way as described earlier in paired comparison methodology. Correlation coefficient between individual score on a statement and total score is also used as the criteria for selection of statements. The higher the r-value for a statement better is the chance of its inclusion in the scale.

(C) Method of Successive Intervals

Each statement is judged according to the degree of favourableness and unfavourableness by the group of judges and the data set are tabulated in the fashion of earlier method of equal appearing (Table-8). This method intends to take into account possible inequalities in the widths of the intervals on the psychological continuum. The scaling problem in the method of successive intervals is to determine estimates of the widths of the intervals making up the psychological continuum from the cumulative proportion distributions for a given set of statements (Edwards, 1943). Firstly, the proportion (P_{ij}) (Table-9) is converted to normal deviates and (Z_{ij}□) table (Table-10) is created. The difference between the successive entries in each of the rows of the table as shown below (Table-11) is calculated to estimate the interval widths. The arithmetic means of the entries in columns are the estimates of the widths of the various intervals as the psychological continuum. Taking arbitrary origin as the upper limit of first interval, the psychological continuum is obtained by cumulating the widths of the various intervals.

Table-8. Judgment data set of successive interval showing the frequencies, cumulative frequencies and cumulative proportions for each statement

Statement	Successive Intervals								
	Unfavourable			Neutral			Favouable		
	1	2	3	4	5	6	7	8	9
Statement1									
f	2	6	16	16	28	52	60	16	4
cf	2	8	24	40	68	120	180	196	200
cp	.010	.040	.120	.200	.340	.600	.900	.980	1.000
Statement 2									
f	4	2	16	20	24	48	66	14	6
cf	4	6	22	42	66	114	180	194	200
cp	.02	.030	.110	.210	.330	.570	.900	.970	1.00
Statement3									
f	2	8	20	24	24	50	56	14	2
cf	2	10	30	54	78	128	184	198	200
cp	.010	.050	.150	.270	.390	.640	.920	.990	1.00

Table-9. Cumulative proportion P_{ij} for 3 statements judged in terms of the method of Successive Intervals (N=200)

Statement	Successive Intervals								
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1	.010	.04	.120	.200	.340	.600	.900	.980	1.000
2	.02	.030	.110	.210	.330	.570	.900	.970	1.000
3	.010	.050	.150	.270	.390	.640	.920	.990	1.000

Table-10. Normal deviates Z_{ij} corresponding to the upper limits of the successive intervals for the data of Table.

Statements	Successive Intervals							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	-	-1.751	-1.175	-.842	-.412	.253	1.282	2.054
2	-2.054	-1.881	-1.227	-.806	-.440	.176	1.282	1.881
3	-	-1.645	-.1.036	-.613	-.279	.358	1.405	-

* Values of P_{ij} less than .02 and more than 0.98 are not considered for obtaining Z_{ij} values

Table-11: Estimate of interval widths

Statements	Successive Intervals						
	2-1	**3-2**	**4-3**	**5-4**	**6-5**	**7-6**	**8-7**
1	-	.576	.333	.430	.159	1.029	.772
2	0.173	.654	.421	.366	.264	1.458	.599
3	-	.609	.423	.334	.637	1.047	-
Sum	0.173	1.839	1.177	1.130	1.060	3.534	1.371
n	1	3	3	3	3	3	2
Mean w.j	.173	.613	.392	.377	.353	1.178	.686
Cumulative Mean w.j	.173	.786	1.178	1.555	1.908	3.086	3.772

This computation provides the width as well as the limit of the particular interval. Using theses figures scale values can be derived by working out the median.

Scale values of the statements

The scale values of the statements may be taken as the medians of the corresponding cumulative proportion distribution of the continuum.

The median may be computed by formula:

$$S_i = l + \left(\frac{50 - \Sigma pb}{pw} \right) x\, \overline{w}.j.$$

S_i = Scale value of the i_{th} stimulus (statement)

L = Lower limit of the interval on the psychological continuum in which the median falls.

Σ pb = the sum of the proportions below the interval in which the median falls.

Pw = the proportion within the interval in which the medium falls

$\overline{w}.j.$ = the width of the interval on the psychological continuum

For statement number 1, the median lies in 6th interval. Therefore, the lower limit (l) will be taken as 1.555. Similarly, substituting the values of , Pb as .340 and Pw as .260 and w as .353, the scale value will be 1.772.

Internal consistency test is performed by calculating the absolute average deviation. Absolute discrepancies over all entries in theoretical proportions and original proportions are summed up and divided by total number of entries to derive the absolute average deviation. The theoretical proportions are worked out with obtained scale values and interval widths. A matrix of scale values (as row) and interval widths (as column) is created and the earlier explained procedure of internal consistency check is followed.

(D) Method of Summated Ratings

Likert's Summated Rating method is most popular among social scientists for scale construction. In this method, initially a large number of statements are collected and edited. Both positive as well as negative statements pertaining to the psychological object are included.

The statements are presented to the subjects with request to respond on 5 or 7-point continuum in terms o their agreement or disagreement and score of 5 to 1 is assigned for positive statement and reversed for a negative statement. After obtaining the response of the requisite number of subject usually numbering 60 to 100, their scores on each statement are summed up. On the basis of total scores of individuals, we take top 20-28 per cent and bottom 20-28 per cent of the subjects in terms of their total score and are labeled as high group and low group respectively. Next step is to find the differences in the mean score on each statement between high and low group of subjects. Statements having high difference in the mean scores between high & low group are selected for the scale. Alternately the t- value can be worked out and as thumb rule a t-value equal to or higher than 1.75 is considered to have discriminatory power and such statements are selected. Likert made the weighting of response categories like strongly agree, agree, uncertain, disagree and strongly disagree simpler by assigning a weight of 4 to a strongly agree response for favourable statements and similarly 0 to strongly disagree response to unfavourable statements. The responses are tabulated for high and low group as shown below (Table-12).

Table- 12. Response of respondents of high group and a low group for a statement

Response Categories	Low Group				High Group			
	x	*f*	*fx*	fx^2	*x*	*f*	*fx*	fx^2
Strongly agree	4	2	8	32	4	16	60	256
Agree	3	6	18	54	3	22	60	198
Uncertain	2	22	44	88	2	8	20	32
Disagree	1	12	12	12	1	3	3	3
Strongly disagree	0	8	0	0	0	1	0	0
Sums		50	82	186		50	143	489

- X = weight of category, f=-frequency of judgment,
- t-value for each statement is calculated with the following formula:

$$t = \frac{\bar{x}_H - \bar{x}_L}{\sqrt{\frac{\Sigma(x_H - \bar{x}_H)^2 + \Sigma(x_L - \bar{x}_L)^2}{n(n-1)}}}$$

H= High group

L= Low group

$t \geq 1.75$

For the above set of data, t works out to be 5.08, which is much higher than the table value i.e. 1.75. Hence the high and low groups differ significantly.

Selection of statements: The statements are arranged in rank order according to their t-values and about 20-25 statements with the largest t-value are selected for the attitude scale.

Whatever the method for selection of statement it is important to include almost equal number of positive and negative statements with high power of discriminability and the total scale should comprise 20-30 statements. The scale before use should be tested for reliability on either split half method or test retest method.

Scale analysis: Guttman believed that a genuine scale, capable of legitimate measurement, exists when homogeneity is complete (Guilford, 1987). The scale should measure one factor only and the response of an individual could be predicted from his score on the scale. Guttman's technique and scale discrimination technique of Edwards and Kilpatrick are used for determining whether a set of statements forms a unidimensional scale.

Every method of scale construction as described above has some advantages and limitations. Selection of any method for scale construction depends upon the nature of task. The Thurstone scale is most appropriate and reliable if the scale is measuring a single attitude and not a complex attitude. However, the process of constructing scale according to Thurstone method is cumbersome. Comparatively, Likert method is easy. The range of responses provided to the statements in this scale offers more precise information about the respondent's disposition to the stimulus i.e. favourableness –unfavourableness or agreement- disagreement.

References:

a) Edwards Allen L. (1969) Techniques of attitude scale constriction. Vakils, Feffer and Simons Pvt. Ltd., Bombay , India.

b) Kerlinger Fred N (1995) Foundations of Behavioural Research IIIrd Ed. Prism Books Rt. Ltd. Bangalore, India

c) Guilford, J.P. (1987). Psychometric methods. Tata Mc Graw –Hill Publishing Co.Ltd, New Delhi

d) Wilkinson T.S. and Bhandarkar, P.L.1984.Methodology and Techniques of Social Research. Himalaya Publishing House. Mumbai.

Chapter -II

Levels of MEASUREMENT

Name of the Experiment: **Measurement in Social Research**

Objective

- To understand the concept and purpose of measurement in social and behavioral reserach
- To learn about the levels of measurement.

Description

Measurement is the assignment of numerals to objects or events according to rules. A numeral is a symbol of the form: 1, 2, 3 ……or I, II, III……. It has no quantitative meaning. Unless we give it such a meaning it is simply a symbol of any special kind.

Postulate of measurement

1. Either $a=b$ or $a \neq b$
2. If $a=b$, then $b=a$
3. If $a=b$, and $b=c$ then $a=c$
4. If $a>b$, then $b>a$
5. If $a>b$ and $b>c$, then $a>c$
6. If $a = p$ and $b>0$, then $a+b>p$
7. $a + b = b + a$
8. If $a = p$ and $b = q$, then $a + b = p + q$
9. $(a + b) + c= a+(b + c)$

There are four general levels of measurement viz., Nominal, Ordinal, Interval, and Ratio.

- **Nominal Measurement**

  It is the lowest level of measurement. The numbers assigned to objects are numerical without having a number meaning e.g. Pele's Jersey number 10, car plate number, PIN code number etc. These numbers can not be ordered or added. They are just labels or used as names without any quantification connotation. The variables such as gender, family type, occupation, caste, etc are measured at nominal level.

- **Ordinal Measurement**

  It provides rank-ordering of a set of objects on an operationally defined characteristic or property. Ordinal numbers indicate rank order and nothing more. The number does not indicate absolute quantities, nor do they indicate that the intervals between the numbers are equal. Rank-order scales are not equal-interval scales, nor do they have absolute zero points. The variables such as education, motivation, attitude, perception, etc are measured at ordinal level. In fact, almost all behvioural constructs are measured at ordinal level.

- **Interval Measurement**

  Interval or equal-interval scale possesses the characteristics of nominal and ordinal scales, especially the rank-order characteristic. In addition, numerically equal distances on interval scales represent equal distances in the property being measured. Computation is done with interval widths or distances.

- **Ratio Measurement**

  It is the highest level of measurement. It possesses the characteristics of nominal, ordinal and interval scales and also has an absolute zero that has empirical meaning. Since there is an absolute zero, all arithmetic operations are possible. The variable such as age, income, size of holding, family size, etc are measured at ratio level.

Note: Most of the instruments in behavioral science belong to ordinal level of measurement but with assumption of equality of interval, researchers could use higher level of statistical operations.

ASSIGNMENTS

- Collect the data of socio-economic profile of farmers from any thesis and comment on the nature of data and classify them as nominal, ordinal, interval and ratio data.
- Collect some scales from theses and comment on their level vis-à-vis quantification and analytical tools employed by the researchers.

METHOD OF PAIRED COMPARISON

Name of the Experiment: **Development of scale with paired comparison method**

Objective

- To understand the concept and method of paired comparison.
- To learn about the Techniques of items selection, computation of scale value and test of significance.

Description

The law of comparative judgment developed by Thurstone in 1920's provided a rationale for the ordering of stimuli along a psychological continuum. The law assumes that for a given stimulus i there is associated a most frequently aroused or modal discriminal process on a psychological continuum. The scale separation of the modal discriminal process $\bar{s}_i$ and $\bar{s}_j$, as the psychological continuum is taken as the function of the proportion of judgement i greater than j.

Methodology

An empirical frequency corresponding to the number of times that i is judged to be more favourable than j is obtained.

$P_{ij} = f_{ij}/N$

Where, P_{ij} is the proportion of times *i* is judged greater than *j*.

- The values of P_{ij} are expressed as unit normal deviates Z_{ij} by means of transformation.
- The selected statements in pairs $\left\{\frac{n(n-1)}{2}\right\}$ are given to a group of 50 to 100 subjects for their comparative judgements as to which of each pair is the more favorable. The original data consist of the frequencies corresponding to the number of times that each stimulus or statement is judged more favourable than every other statement.
- A table of frequency matrix (f_{ij}) is prepared.
- The frequency matrix is converted into proportion matrix (p_{ij}) with formula $P_{ij} = 1/N\ f_{ij}$
- Corresponding to the P_{ij} entries, normal deviates are obtained from Table (appendix) and Z_{ij} matrix table is prepared.
- Arithmetic mean of the entries in the column of the Z matrix is calculated, as shown in table below.
- The scale values of the stimuli are obtained in terms of their deviation from the mean of all the scale values.
- Since the origin is arbitrary a constant is added to make all the deviations positive. A convenient constant to add is the absolute scale value of the stimulus with the largest negative deviation.

Table-13: P-matrix for judgment

Statements	1	2	3	4
1	.500	.691	.798	.851
2	.309	.500	.543	.574
3	.202	.457	.500	.521
4	.149	.426	.479	.500

Table-14 : Z-matrix for judgment

Statements	1	2	3	4
1	.000	.499	.834	1.041
2	-.499	.000	.108	.187
3	-.834	-.108	.000	.053
4	-1.041	-.187	-.053	.000
Sum	-2.374	.204	.889	1.281
Mean	-0.594	0.05	.222	.320
Means +0.594	0.000	0.644	.816	.914

The Internal Consistency Check

This check involves determining how well our deserved or empirical proportions p_{ij} agree with those to be expected in terms of our derived scale values.

Steps:

- Setup the table where, the rows and columns are bounded by scale value as shown below.

Table-15: Difference between P_{ij} - P_{ij}'

Statement	Scale values	1 .000	2 .644	3 .816	4 .914
1	.000	-	-	-	-
2	.644	-.644	.000	-	-
3	.816	-.816	-.171	-	-
4	.914	-.914	-.269	-.098	-

- Obtain a matrix z' of theoretical normal deviates corresponding to the scale separations of the statements.
- Subtracting the entries at the left of the table from the scale value for stimulus 1 at the top of column (1), we obtain the theoretical normal deviates Z_{ij} entered in the first column of the table. Similarly the Z_{ij} ' values for other stimuli are obtained.
- With the Z_{ij} values the corresponding theoretical p_{ij} are obtained.
- The entries in the P' matrix the corresponding entries in the P matrix is subtracted to obtain the discrepancies between theoretical and empirical proportions.
- Absolute average discrepancy (AD) is calculated by the formula

$$AD = \frac{.\Sigma\left|pij - pij'\right|}{\frac{n(n-1)}{2.}}$$

SOLVED EXAMPLE: DETERMINATION OF SCALE VALUE USING PAIRED COMPARISON TECHNIQUE

Given below is the frequency matrix (F_{ij}) of judgment by 100 judges. The frequency matrix is converted into proportion matrix (P_{ij}) and normal deviate matrix (Z_{ij}) and scale values are calculated as explained earlier.

Table-16: F-matrix (F_{ij}) giving the frequency with which the column statement is judged more favourable than the row stimulus

N=100

Statements	1	2	3	4	5	6	7
1	50	65	75	80	75	86	88
2	35	50	51	54	62	68	81
3	25	49	50	49	59	60	63
4	20	46	51	50	49	63	67
5	25	38	41	51	50	51	55
6	14	32	40	37	49	50	57
7	12	19	37	33	45	43	50
Sum	181	299	345	354	389	421	461

Table-17: P-Matrix (P_{ij}) corresponding to above F--matrix

Statements	1	2	3	4	5	6	7
1	.50	.65	.75	.80	.75	.86	.88
2	.35	.50	.51	.54	.62	.68	.81
3	.25	.49	.50	.49	.59	.60	.63
4	.20	.46	.51	.50	.49	.63	.67
5	.25	.38	.41	.51	.50	.51	.55
6	.14	.32	.40	.37	.49	.50	.57
7	.12	.19	.37	.33	.45	.43	.50

Table-18: Z-Matrix (Z_{ij}) corresponding to above P--matrix

Statement	1	2	3	4	5	6	7
1	0	0.385	0.674	0.842	0.674	1.08	1.175
2	-0.385	0	0.025	0.1	0.305	0.412	0.876
3	-0.674	-0.025	0	-0.025	0.228	0.253	0.332
4	-0.842	-0.1	0.025	0	-0.025	0.332	0.44
5	-0.674	-0.305	-0.228	0.025	0	0.025	0.126
6	-1.08	-0.468	-0.253	-0.332	-0.025	0	0.176
7	-1.175	-1.08	-0.332	-0.44	-0.126	-0.176	0
Sum	-4.83	-1.593	-0.089	0.17	1.031	1.926	3.125
Average	-0.69	-0.22757	-0.01271	0.024286	0.147286	0.275143	0.446429
Mean+.690	0.000	0.462	0.677	0.714	0.837	0.965	1.136

- The last row shows the scale values of respective statements

Chapter IV

SIGNIFICANCE TESTS FOR PAIRED COMPARISON JUDGEMENTS

Name of the Experiment: **Significance tests for paired comparison judgements**

Objective

- To understand the concept of Significance tests for paired comparison judgements
- To learn about the methods to compute Significance tests

Description

The significance tests help to measure the discrepancies between the observed and theoretical proportions of the judgements.

(a) Test of significance for the case V model.

The X^2 test to determine whether the observed P_{ij} and theoretical P_{ij} values are in accord with each other is based upon a transformation of both the theoretical and observed proportions.

$$\theta = arc \sin \sqrt{p}$$

Steps

- Develop the table with values of θ corresponding to the empirical proportions p_{ij}.
- Develop the table with values of θ corresponding to the theoretical proportions p_{ij}.
- Calculate x^2 with the following formula

$$x^2 = \frac{\Sigma(\theta - \theta')^2}{821/N}$$

$$df = \frac{(n-1)(n-2)}{2}$$

Solved example:

A matrix of scale values obtained in Table-18 is created as below in Table-19 and each row entry is subtracted from column entry. For example, the first column data is obtained by subtracting the all scale values of 7 statements from 0.000. Similarly, the data for the column 2, data could be obtained by subtracting the all scale values of 7 statements from 4.62. These data are theoretical normal deviates (Z_{ij}'). In similar manner the data is obtained for all columns. However, only the data below the diagonal are considered for transformation to proportion, which would be theoretical proportion (P_{ij}'). The theoretical proportion (P_{ij}') is shown in Table-20.

Table-19: Theoretical normal deviates (Z_{ij}').

Statement	Scale value	Scale value						
		1	2	3	4	5	6	7
		0.000	**0.462**	**0.677**	**0.714**	**0.837**	**0.965**	**1.136**
1	**0.000**	0.000	-	-	-	-	-	-
2	**0.462**	-0.462	0	-	-	-	-	-
3	**0.677**	-0.677	-0.215	0	-	-	-	-
4	**0.714**	-0.714	-0.252	-0.037	0	-	-	-
5	**0.837**	-0.837	-0.375	-0.16	-0.123	0	-	-
6	**0.965**	-0.965	-0.503	-0.288	-0.251	-0.128	0	-
7	**1.136**	-1.136	-0.674	-0.459	-0.422	-0.299	-0.171	0

Table-20: Theoretical proportion (Pij') corresponding to normal deviates (Zij') obtained in Table-19

Statement	1	2	3	4	5	6	7
1	-	-	-	-	-	-	-
2	.322	-	-	-	-	-	-
3	.240	.415	-	-	-	-	-
4	.238	.400	.485	-	-	-	-
5	.201	.354	.436	.451	-	-	-
6	.167	.307	.387	.401	.449	-	-
7	.128	.250	.323	.336	.383	.432	-

The observed proportion (P_{ij}) and theoretical proportions (P_{ij}') are converted into θ and θ' values respectively as shown in Table-21 and Table-22.

Table-21: θ values corresponding to P-Matrix of table-17.

Statements	1	2	3	4	5	6	7
1	-						
2	36.27	-					
3	30.00	44.43	-				
4	26.56	42.71	45.57	-			
5	30.00	38.06	39.82	45.57	-		
6	21.97	34.45	39.23	37.47	44.43	-	
7	20.27	25.84	37.47	35.06	42.13	40.98	-

Table-22:a θ' values corresponding to Pij' matrix of table-20.

Statements	1	2	3	4	5	6	7
1	-	-					
2	34.57	-					
3	29.33	40.11	-	-	-		-
4	29.20	39.23	44.14	-	-		-
5	26.64	36.51	41.32	42.19	-		-
6	24.12	33.65	38.47	39.29	42.07		-
7	20.96	30.00	34.63	35.43	38.23	41.09	-

Table-22-b: Values of (θ - θ') derived from table 21 and table-22

Statements	1	2	3	4	5	6	7
1	-	-	-	-	-	-	-
2	1.7	-	-	-	-	-	-
3	0.67	4.32	-	-	-	-	-
4	-2.64	3.48	1.43	-	-	-	-
5	3.36	1.55	-1.5	3.38	-		-
6	-2.15	0.8	0.76	-1.82	2.36	-	-
7	-0.69	-4.16	2.84	-0.37	3.9	-0.11	-

$\Sigma(\theta - \theta')^2 = (1.7)^2 + (0.67)^2 + \ldots\ldots\ldots\ldots\ldots\ldots + (-0.11)^2 = 126.42$

$x^2 = 126.42 / (821/N) = 126.42/8.21 = 15.40$

$df = (7-1)*(7-2)/2 = 15$

The probability P for obtaining x^2 value as 15.40 with degree of freedom as 15, will lie between 0.30 to 0.50. It shows that the obtained x^2 value is not significant. It means that the assumptions made for obtaining scale values are tenable.

Identification of Circular Triads and computation of Coefficient of Consistence

Presence of circular triad in the paired judgements reflects inconsistency in judgement. With respect to three statement e.g. i, j, and k, if a subject judges statement i more favourable than j and judges statement j more favourable than k then to reflect consistency the statement i should be judged more favorable than K. If i is judged less favourable than k, these three statements i, j and k form a circular triad. The greater the number of circular triads occurring in the set of comparative judgements of a given subject, the more inconsistent the subject is supposed to be:

For odd number of stimuli, the maximum possible number of circular triads, is obtained by the formula:

$$d = \frac{n^3 - n}{24}$$

Coefficient of consistence, zeta $\xi = 1 - \frac{24d}{n^3 - n}$

- For even number of stimuli, the maximum possible number of circular triads, is obtained by the formula:

$$\xi = 1 - \frac{24d}{n^3 - 4n}$$

Coefficient of consistence, zeta $d = \frac{n^3 - 4n}{24}$

Where,

n = Number of stimuli (statements)
d = Observed number of circular triads for a given subject.
=

$$= \left(\frac{1}{12}\right)(n)(n-1)(2n-1) - \frac{1}{2}\Sigma a^2$$

a = The number of entries in a column

Methodology

- Prepare the table as mentioned below with entries of 1 and 0. When the column stimuli is judged more favourable than the row stimuli 1 is entered in the corresponding cell of the table.
- If the column stimulus is judged less favourable than the row stimulus, entry of 0 is made.

Table-23:Comparative judgment pattern of a judge

Statement	**1**	**2**	**3**	**4**	**5**	**6**	**7**
1	-	1	1	1	1	1	1
2	0	-	1	1	1	1	1
3	0	0	-	1	1	1	1
4	0	0	0	-	1	1	1
5	0	0	0	0	-	1	1
6	0	0	0	0	0	-	1
7	0	0	0	0	0	0	-

Significance test for the Coefficient of Consistency

It is calculated by the formula given below:

$$x^2\left(\frac{8}{n-4}\right)\left(\frac{1}{4}\,{}^{n}C_3 - d + \frac{1}{2}\right) + df$$

d = observed number of circular triads

df= the number of degree of freedom associated with x^2

$$df = \frac{n(n-1)(n-2)}{(n-4)^2}$$

Coefficient of Agreement

Coefficient of Agreement (u) developed by Kendall (1948) provides a means of determining the extent to which a group of judges agree in their comparative judgements.

$$u = \frac{2\tau}{({}^{m}C_2)({}^{m}C_2)} - 1$$

where, $\tau = (\Sigma fij^2 - m\Sigma fij) + ({}^{m}C_2)({}^{n}C_2)$

fij^2 = the sum of the squared fij entries below the diagonal
m = the number of judges
Σ fij= the sum of the entries below the diagonal
n = the number of stimuli (statements)

The value of u varies from 1.00 to -1.00. If the number of judges is even, than the minimum value of u is –1(m-1). If the number of judges is odd, then the minimum value of u is –1/m

The smaller the value of u, the greater is the departure from complete agreement.

x^2 test for the coefficient of agreement

$$x^2\left[\frac{4}{m-2}\right]\left[\tau-\frac{1}{2}\left({}^nC_2\right)\left({}^mC_2\right)\left(\frac{m-3}{m-2}\right)\right]$$

$$df = \left({}^nC_2\right)\frac{m(m-1)}{(m-2)^2}$$

DETERMINATION OF CIRCULAR TRIADS AND COEFFICIENT OF CONSISTENCY IN COMPARATIVE JUDGEMENT

1. Suppose the number of statements n= 9

∴ Maximum number of circular triads that can occur = $(n^3-n)/24$

= (729–9)/24 = 30

Coefficient of consistency ξ for n = 9

$$\xi = \frac{1-24\text{ d}}{n^3-n}$$

Number of circular triads (d)

$= (1/12)\ (n)\ (n-1)\ (2n-1) - ½ \sum a^2$

a = sum of entries in a given column.

χ^2 test of significance:

$\chi^2 = (8/n-4)\ (1/4\ nC_3 - d + 1/2) + df.$

$$df = \frac{n(n-1)\ (n-2)}{(n-4)^2}$$

Table-24: Response pattern of a judge

Statement	1	2	3	4	5	6	7	8	9
1	-	1	0	1	1	1	1	1	1
2	0	-	1	1	0	1	1	0	1
3	1	0	-	0	1	0	1	0	1
4	0	0	1	-	0	1	1	0	1
5	0	1	0	1	-	1	0	1	0
6	0	0	1	0	0	-	1	0	0
7	0	0	0	0	1	0	-	1	0
8	0	1	1	1	0	1	0	-	1
9	0	0	0	0	1	0	1	0	-
a	**1**	**3**	**4**	**4**	**4**	**5**	**6**	**3**	**5**
a^2	**1**	**9**	**16**	**16**	**16**	**25**	**36**	**9**	**25**

$\sum a^2 = 153$

1) $d = 1/12 \times 9 \times 8 \times 17 - ½ \times 153$

   $= 120 - 76.5 \quad = 25.5 \sim 26$

2) $\xi = (1-24\times26)/ 729\text{-}9$

   $= 1-0.867 = 0.133$

3) $\chi^2 = (8/9\text{-}4)\ (1/4)\ pC_3 - 26 + 1/2) + 20$

   $= (1.6)\ (21 - 26.5) + 20$

   $= -8.8 + 20 = 11.2$

P = .98 corresponding value of $\chi^2 = 11.2$

$\therefore$ the probability of obtaining a value of $d \geq 26$

$= 1- .98 = 0.2$

It means the number of circular triad committed by the judge is not significant.

DETERMINATION OF THE COEFFICIENT OF AGREEMENT IN COMPARATIVE JUDGEMENT

Coefficient of agreement $u = \dfrac{2T}{(mC_2)\ (nC_2)} - 1$

$T = (\sum f_{ij}^2 - m \sum f_{ij}) + (mC_2)\ (nC_2)$

m= number of judges (96)

n= number of statements (7)

Table-25: F –Matrix

Statement	1	2	3	4	5	6	7
1	–						
2	31	–					
3	21	45	–				
4	16	42	47	–			
5	21	34	37	47	–		
6	10	28	36	33	45	–	
7	8	15	33	29	41	39	–
fij	107	164	149	109	86	39	$\sum f_{ij}$=654
fij^2	2263	5954	5715	4139	3706	1521	$\sum f_{ij}^2$=23298

T = (23298 – 96 ×654) + (4560) × (21)

= - 39489 +95760

= 56274

$mC^2 = \dfrac{96!}{}$

$$= \frac{\frac{94! \times 2!}{96\times95}}{2}$$

$= 4560$

$$nC_2 = \frac{7\times6}{2}$$

$= 21$

$$u = \frac{2\times56274}{4560\times21} - 1$$

$= 1.175 - 1 = 0.175$

The greater the departure from complete agreement the smaller the value of 'u'

χ^2 test for the coefficient of agreement:

χ^2 = [4/m-2] [T -1/2 (nC_2) (mC_2) (m-3/m-2)]

= [4/96-2] [56274 – ½ × 21 × 4560 × 09894]

= 0.043 (56274 – 47372)

= 0.043 × 8902 = 382.79

df = (n C_2) m(m-1)/(m=2)^2

= 21 [96×95/94^2] = 21.67 ~ 22

The value of χ^2 = 382.79 at 22 df is highly significant since the table value of χ^2 at 22 df and .01 level of significance = 40.289, Therefore, it shows that the 96 judges have shown significant agreement in their comparative judgment.

ASSIGNMENTS

Q1. What is circular triad?

Data on comparative judgment of an expert for seven statements are presented in table below. Find out the number of circular triad and maximum possible number of circular triads. Examine the consistency of judgment of the experts.

Response	Statements						
	1	2	3	4	5	6	7
1	-	0	1	1	1	0	0
2	1	-	1	1	0	1	0
3	0	0	-	0	1	0	1
4	0	0	1	-	0	1	0
5	0	1	0	1	-	0	1
6	1	0	1	0	1	-	0
7	1	1	0	1	0	1	-

DETERMINATION OF SCALE VALUE OF STATEMENTS USING CASE V (INCOMPLETE DATA) MODEL

The method of Paired Comparison: Case V: Incomplete data

- When the extreme values of P_{ij} (.02 ≥ p_{ij} ≥ .98) are ignored there will be missing entries in the Z matrix. In such a situation the method of calculating the scale values will be different.
- Beginning from the first column the entries in column is subtracted from the corresponding entries in successive columns and thus a new matrix is prepared.
- The scale value are calculated by formula: $\bar{s}_i - \bar{s}_{(i-1)} = D_{i(i-1)}$
- Dividing the sums of the column entries by the corresponding number of entries we obtain the value of $\overline{D}_{i(i-1)} = \bar{s}_i - \bar{s}_{(i-1)}$

Guidelines for preparing Tables and Computation of Scale values

P Matrix

Statements	1	2	3
1	.500	.923	.923
2	.077	.500	.526
3	.077	.474	.500

Z Matrix

Statements	1	2	3
1	.000	1.426	1.635
2	-1.426	.065	.616
3	-1.426	.000	.292

Matrix of successive differences

Statements	2-1	3-2	4-3
1	1.426	.000	.209
2	1.426	.065	.551
3	1.361	.065	.292
Sum			
n			
Mean			
Scale Values	S_2	S_3	S_4
S_1= .000			

Solved example

Given below is the frequency matrix. It can be observed that the sum of frequency of statement 4 is less than statement 3. In order to attain the scale values in ascending order the data entry should be such that sum of the scores are in ascending order. Therefore, it placement of score of statement 4 should be kept before the statement 3 as shown in Table-26

Table-26:F-matrix

N=96

Statements	1	2	3	4	5	6	7
1	48	65	75	92	96	90	96
2	31	48	88	56	68	92	86
3	21	8	48	51	61	62	65
4	4	40	45	48	51	65	69
5	0	28	35	45	48	53	57
6	6	4	34	31	43	48	59
7	0	10	31	27	39	37	48
Sum	110	203	356	350	406	447	480

Table-27: F-matrix with change of place of statement 4 and keeping it before the statement3.

Statements	1	2	3	4	5	6	7
1	48	65	92	75	96	90	96
2	31	48	56	88	68	92	86
3	4	40	48	45	61	62	65
4	21	8	51	48	51	65	69
5	0	28	45	35	48	53	57
6	6	4	31	34	43	48	59
7	0	10	27	31	39	37	48

Table-28:P-matrix

Statements	1	2	3	4	5	6	7
1	.500	.677	.958	.781	**1.000**	.938	**1.000**
2	.323	.500	.583	.917	.708	.958	.896
3	.042	.417	.500	.469	.583	.677	.719
4	.219	.083	.531	.500	.635	.646	.677
5	**.000**	.292	.469	.365	.500	.552	.594
6	.063	.042	.323	.354	.448	.500	.615
7	**.000**	.104	.281	.323	.406	.385	.500
Sum	1.147	2.115	3.645	3.709	4.28	4.656	5.001

*P_{ij} ($.02 \geq p_{ij} \geq .98$) are ignored

Table-29:Z-matrix

Statements	1	2	3	4	5	6	7
1	.000	.459	1.728	.776	—	1.538	—
2	-.459	.000	.210	1.385	.548	1.728	1.259
3	-1.728	-.210	.000	-.078	.210	.459	.580
4	-.776	-1.385	.078	.000	.345	.375	.459
5	—	-.584	-.078	-.345	.000	.131	.238
6	-1.530	-1.728	-.459	-.375	-.131	.000	.292
7	—	-1.259	-.580	-.459	-.238	-.292	.000

Table-30: Matrix of successive difference of column entries

Statements	Column differences					
	2-1	3-2	4-3	5-4	6-5	7-6
1	0.459	1.269	-0.952	-	-	-
2	0.459	0.21	1.175	-0.837	1.18	-0.469
3	1.518	0.21	-0.078	0.288	0.249	0.121
4	-0.609	1.463	-0.078	0.345	0.03	0.084
5	-	0.506	-0.267	0.345	0.131	0.107
6	-0.198	1.269	0.084	0.244	0.131	0.292
7	-	0.679	0.121	0.221	-0.054	0.292
Sums	1.629	5.606	0.005	0.606	1.667	0.427
n	5	7	7	6	7	6
Mean	0.326	0.801	0.001	0.101	0.238	0.071

Scale values:

S_1	S_2	S_3	S_4	S_5	S_6	S_7
.000	**.326**	**1.127**	**1.128**	**1.229**	**1.467**	**1.538**

DETERMINATION OF SCALE VALUE USING CASE III MODEL OF COMPARATIVE JUDGEMENT IN PAIRED COMPARISON TECHNIQUE

Working out scale values of statements without assumption of equality of discriminal dispersion (σ_i) is referred as CASE III model of Thurstone's Law of Comparative judgment. Therefore, the process in CASE III will be to first generate the F-matrix, P-matrix and Z matrix as in CASE V model. Later the Z matrix is converted in to new matrix by multiplying with square root of sum of standard deviations of respective pair statements *i* and *j*.

In this model, firstly discriminal dispersion (σ_i) of statements is worked out. The new Z matrix is created with formula as given below:

Z_{ij}* SQRT($\sigma_i^2 + \sigma_j^2$)

Eg. First entry in Z matrix will be Z_{11} * SQRT($\sigma_1^2 + \sigma_1^2$)

Second entry will be Z_{12} * SQRT($\sigma_1^2 + \sigma_2^2$)

Working out the scale value using CASE III model for data of Table-17.

The discriminal dispersion (σ_i) of statements is calculated by following formula:

$\sigma_i = a(1/V_i)-1$; where $a = 2n/\sum(1/V_i)$; n is number of statements

$V_i^2 = \sum (Z_{ij} - Z_i)^2/n$

The discriminal dispersion (σ_i) of 7 statements can be calculated as below:

$\sigma_1 = a(1/V_1)-1$

$\sigma_2 = a(1/V_2)-1$

$\sigma_3 = a(1/V_3)-1$

$\sigma_4 = a(1/V_4)-1$

$\sigma_5 = a(1/V_5)-1$

$\sigma_6 = a(1/V_6)-1$

$\sigma_7 = a(1/V_7)-1$

Given below is Z matrix (Table-18).

Z-matrix

Statement	1	2	3	4	5	6	7
1	0	0.385	0.674	0.842	0.674	1.08	1.175
2	-0.385	0	0.025	0.1	0.305	0.412	0.876
3	-0.674	-0.025	0	-0.025	0.228	0.253	0.332
4	-0.842	-0.1	0.025	0	-0.025	0.332	0.44
5	-0.674	-0.305	-0.228	0.025	0	0.025	0.126
6	-1.08	-0.468	-0.253	-0.332	-0.025	0	0.176
7	-1.175	-1.08	-0.332	-0.44	-0.126	-0.176	0
$\sum Z_{ij}^2$	4.313	1.637	0.682	1.024	0.616	1.542	2.499
$\sum Z_{ij}$	-4.830	-1.593	-0.089	0.170	1.031	1.926	3.125
$(\sum Z_{ij})^2/n$	3.333	0.363	0.001	0.004	0.152	0.530	1.395
$\sum Z_{ij}^2 - (\sum Z_{ij})^2/n$	0.980	1.275	0.681	1.020	0.465	1.012	1.104
V^2	0.140	0.182	0.097	0.146	0.066	0.145	0.158
V	0.374	0.427	0.312	0.382	0.258	0.380	0.397
1/V	2.673	2.343	3.207	2.620	3.882	2.630	2.519

Using the above formula, we can work out the standard deviations

Statement	Standard deviation	Value
1	σ_1	0.883
2	σ_2	0.651
3	σ_3	1.259
4	σ_4	0.846
5	σ_5	1.735
6	σ_6	0.853
7	σ_7	0.774

Now we need to convert the Z matrix in a new matrix by multiplying each Z_{ij} with SQRT($\sigma_i^2 + \sigma_j^2$). After that, scale values are worked out like CASE V model.

Chapter -V

METHOD OF EQUAL-APPEARING INTERVALS

Name of the Experiment: **Development of scale with the method of equal-appearing intervals**

Objective

- To understand the concept of the method of equal-appearing intervals.
- To learn about the methods to compute scale values.

Description

The method of equal-appearing interval unlike paired comparison method requires each subject to make only one comparative judgement for each statement as a result a fairly large number of statements could be handled with ease.

Each subject is asked to judge the degree of favourableness or unfavourableness of feeling expressed by each statement in terms of 11 intervals. Only the first, middle and the last card are labeled (most favourable, neutral and most unfavourable, respectively)

The judgements are put in tabular format as shown below. For each statement there are 3 rows. First row shows the frequency with which the statement was placed in each of the 11 categories, second given the proportion of judgement i.e. frequency divided by number of total number of judges, while the third row gives the cumulative proportions.

Table-31: Judgement obtained by the method of equal-appearing intervals

Statements	Sorting categories											Scale Value	Q Value
	A	B	C	D	E	F	G	H	I	J	K		
	1	2	3	4	5	6	7	8	9	10	11		
f	2	2	6	2	6	65	64	26	18	8	4	6.8	1.7
p	.01	.01	.03	.01	.03	.31	.32	.13	.09	.04	.02		
cp	.01	.02	.05	.06	.09	.40	.72	.85	.94	.98	1.00		

Scale value(s)

The median of the distribution of judgements for each statement is taken as the scale value of the statement, calculated by the formula given below:

$$S = l + \left(\frac{.50 - \Sigma pb}{pw} \right) x \text{ i}$$

S = the median or scale value of the statement
l = Lower limit of the interval in which the median falls.
Σ pb = the sum of the proportions below the interval in which the median falls.
pw = the proportion within the interval in which the medium falls
i = the width of the interval and is assured to be equal to 1.0

Inter-quartile range (Q)

Inter-quartile range (Q) is used as a measure of the variation of the distribution of judgements for a given statement. It contains the middle 50 per cent of the judgements. It is the measure of the spread of the middle 50 per cent of the judgements.

$$Q = C_{75} - C_{25}$$

$$C_{75} = l + \left(\frac{.75 - \Sigma pb}{pw}\right) x \text{ i}$$

$$C_{25} = l + \left(\frac{.25 - \Sigma pb}{pw}\right) x \text{ i}$$

Note : Large values of Q show disagreement among judges. It is an indication that a statement is ambiguous. The statements with larger Q values are not included in the scale.

SOLVED EXAMPLE

DETERMINATION OF SCALE VALUES AND Q-VALUE IN METHOD OF EQUAL APPEARING INTERVAL

Table-32: Judgment obtained for 5 statements over 11 equal-appearing intervals

Statement		SCORING CATEGORIES										
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
		A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**
1	F	2	2	6	2	6	62	64	26	18	10	2
	P	.01	.01	.03	.01	.03	.31	.32	.13	.09	.05	.01
	CP	.01	.02	.05	.06	.09	.40	.72	.85	.94	.99	1.00
2	F	3	2	5	1	7	60	66	24	22	8	2
	P	.015	.01	.025	.005	.035	.3	.33	.12	.11	.04	.01
	CP	.015	.025	.05	.055	.095	.395	.725	.845	.855	.995	1.00
3	F	0	0	0	10	38	30	50	26	28	14	4
	P	0	0	0	.05	.19	.15	.25	.13	.14	.07	.02
	CP	0	0	0	.05	.24	.36	.64	.77	.91	.98	1.00
4	F	2	0	0	12	36	38	42	24	30	14	2
	P	.01	0	0	.06	.18	.19	.21	.12	.15	.07	.01
	CP	.01	.01	.01	.07	.25	.44	.65	.77	.92	.99	1.00
5	F	0	0	0	2	6	8	26	44	60	40	14
	P	0	0	0	.01	.03	.04	.13	.22	.3	.2	.07
	CP	0	0	0	.01	.04	.08	.21	.43	.73	.93	1.00

Scale value : $S = l + (.50 - \sum bp/pw) \times i$

Inter quartile range: $Q = C_{75} - C_{25}$

Statements	Scale value	Q-value
1	6.8	1.7
2	6.8	1.7
3	6.9	2.8
4	6.8	2.8
5	8.7	1.9

Statements 1,2 and 5 can be related for the construction of scale due to the high scale value as well as the low value compared to the test two statements.

ASSIGNMENTS

Q1. Given below is summary of frequencies with which the respective statements were placed in each of categories by judges. Please calculate the scale values as well as Q value for the statements and select 3 best statements for the scale.

Statements	**Scoring categories**										
	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**
1	4	4	6	2	6	58	60	26	20	8	6
2	2	6	4	3	11	60	62	26	22	8	2
3	1	4	6	14	28	30	35	42	22	12	6
4	2	0	0	7	46	38	42	22	32	9	2
5	0	0	0	4	6	8	26	42	61	41	12

Chapter V

METHOD OF SUCCESSIVE INTERVALS

Name of the Experiment: **Development of scale with the method of Successive Intervals**

Objective

- To understand the concept of the method Successive Intervals
- To learn about the methods to compute scale values and internal consistency test.

Description

This method intends to take into account possible inequalities in the widths of the intervals on the psychological continuum. The scaling problem in the method of successive intervals is to determine estimates of the widths of the intervals making up the psychological continuum from the cumulative proportion distributions for a given set of statements.

- The proportion (Pij) is converted to normal deviates and (Pij□) table is created.
- The difference between the successive entries in each of the rows of above table provides additional estimates of various interval widths.
- The arithmetic means of the entries in columns are the estimates of the widths of the various intervals as the psychological continuum.
- Taking arbitrary origin the upper limit of first interval the psychological continuum is obtained by cumulating the widths of the various intervals.

Scale values of the statements

The scale values of the statements may be taken as the medians of the corresponding cumulative proportion distribution of the continuum.

The median may be computed by formula:

$$S_i = l + \left(\frac{50 - \Sigma pb}{pw} \right) x\, \overline{w}.j.$$

S_i = Scale value of the i_{th} stimulus (statement)

L = Lower limit of the interval on the psychological continuum in which the median falls.

Σ pb = the sum of the proportions below the interval in which the median falls.

Pw = the proportion within the interval in which the medium falls

$\overline{w}.j.$ = the width of the interval on the psychological continuum

Internal Consistency Test

- Using the parameters as n scale values and k-2 interval widths on the psychological continuum, a theoretical cumulative distribution for each statement is generated.
- Subtracting the scale value form each statement from the cumulative interval width, a matrix of theoretical normal deviates is obtained.
- These normal deviates will be the boundaries of the successive intervals.
- From these normal deviates corresponding theoretical proportions are obtained.
- The absolute discrepancies over all entries in theoretical proportions and original proportions are summed up and divided by total number of entries to derive the absolute average deviation.
- An average error of 0.0025 for 10 stimuli for 9 categories and 0.021 for 17 stimuli into 10 categories have been reported fairly typical in the method of successive interval.

Guidelines for tabulation and computation

Arrangement of successive interval data showing the frequencies, cumulative frequencies and cumulative proportions for each statement

Statement	Successive Intervals								
	Unfavourable			Neutral			favouable		
	1	2	3	4	5	6	7	8	9
f	2	4	12	12	26	52	60	26	6
cf	2	6	18	30	56	108	168	194	200
cp	.010	.030	.090	.150	.280	.540	.840	.970	1.000

Cumulative proportion p_{ij} for 3 statements judged in terms of the method of Successive Intervals (N=200)

Statement	Successive Intervals								
	1	2	3	4	5	6	7	8	9
1	.010	.030	.090	.150	.280	.540	.840	.970	1.000
2	-	-	-	.010	.030	.130	.570	.940	1.000
3	.010	.070	.230	.370	.550	.810	.930	.980	1.000

Normal deviates Z_{ij} corresponding to the upper limits of the successive intervals for the data of Table.

Statements	Successive Intervals							
	1	2	3	4	5	6	7	8
1	-	-1.881	-1.341	-1.036	-.583	.100	.994	1.881
2	-	-	-	-	-1.881	-1.126	.176	1.555
3	-	-1.476	-.739	-.332	.126	.878	1.476	2.054

Estimate of interval widths

Statements	Successive Intervals		
	2-1	3-2	4-3
1	-	-1.881	-1.341
2	-	-	-
3	-	-1.476	-.739

SOLVED EXAMPLE: SUCCESSIVE INTERVAL TECHNIQUE

- **Estimation of interval width**

Table-33: Cumulative proportions 'P_{ij}' for 15 statements

Stimuli	Successive Intervals						
	1	2	3	4	5	6	7
A	.000	.010	.010	.050	.290	.810	1.000
B	.010	.030	.080	.160	.410	.740	1.000
C	.010	.010	.020	.080	.260	.630	1.000
D	.052	.104	.219	.427	.656	.875	1.000
E	.000	.000	.021	.155	.505	.784	1.000
F	.040	.050	.110	.350	.710	.950	1.000
G	.010	.040	.110	.300	.680	.890	1.000
H	.011	.022	.088	.363	.648	.890	1.000
I	.000	.030	.110	.360	.630	.870	1.000
J	.060	.142	.290	.560	.790	.960	1.000
K	.010	.042	.146	.490	.750	.938	1.000
L	.010	.030	.091	.303	.606	.788	1.000
M	.010	.020	.082	.347	.571	.908	1.000
N	.010	.041	.071	.235	.551	.837	1.000
O	.010	.030	.061	.172	.333	.646	1.000
□	.243	.599	1.509	4.352	8.390	12.516	15.000

Table-34: Z-Matrix

Statement	1	2	3	4	5	6
A	-	-		-1.645	-0.553	0.878
B	-	-.1.881	-1.405	-0.994	-0.228	0.643
C	-	-	-2.054	-1.405	-0.643	0.332
D	-1.626	-1.259	-0.776	-0.184	0.402	1.15
E	-	-	-2.034	-1.015	0.013	0.786
F	-1.751	-1.645	-1.227	-0.385	0.553	1.645
G	-	-1.751	-1.227	-0.524	0.468	1.227
H	-	-2.014	-1.353	-0.35	0.38	1.227
I	-	-1.881	-1.227	-0.358	0.332	1.126
J	-1.555	-1.071	-0.553	0.151	0.806	1.751
K	-	-1.728	-1.054	-0.025	0.674	1.538
L	-	-1.881	-1.335	-0.516	0.269	0.8
M	-	-2.054	-1.392	-0.393	0.179	1.329
N	-	-1.739	-1.461	-0.722	0.128	0.982
O	-	-1.881	-1.546	-0.946	-0.432	0.375

Values of Pij <0.2 and >0.98 were eliminated

Table-35:Estimate of interval width

Statement	2-1	3-2	4-3	5-4	6-5
A				1.092	1.431
B		-3.286	0.411	0.766	0.871
C			0.649	0.762	0.975
D	0.367	0.483	0.592	0.586	0.748
E			1.019	1.028	0.773
F	0.106	0.418	0.842	0.938	1.092
G		0.524	0.703	0.992	0.759
H		0.661	1.003	0.73	0.847
I		0.654	0.869	0.69	0.794
J	0.484	0.518	0.704	0.655	0.945
K		0.674	1.029	0.699	0.864
L		0.546	0.819	0.785	0.531
M		0.662	0.999	0.572	1.15
N		0.278	0.739	0.85	0.854
O		0.335	0.6	0.514	0.807
sum	0.957	2.467	10.978	11.659	13.441
n	3	12	14	15	15
Wij	0.319	0.206	0.784	0.777	0.896
Cum Wij	0.319	0.525	1.103	1.096	1.215

Working out The scale value could be worked out with following formula:

$$S_i = l + \left(\frac{0.50 - \Sigma pb}{pw} \right) x\, \overline{w}.j.$$

The obtained scale value of each statement is as below:

Table-36: Scale value

Statement	L=lower limit	$(0.50 - \Sigma pb)$	pw	$\overline{w}.j$	Scale value
A	1.096	0.21	0.52	0.896	1.458
B	1.096	0.09	0.33	0.896	1.340
C	1.096	0.24	0.37	0.896	1.677
D	1.103	0.073	0.229	0.896	1.389
E	1.103	0.345	0.35	0.777	1.869
F	1.103	0.15	0.36	0.777	1.427
G	1.103	0.2	0.38	0.777	1.512
H	1.103	0.137	0.285	0.777	1.477
I	1.103	0.14	0.27	0.777	1.506
J	0.525	0.21	0.27	0.777	1.129
K	1.103	0.01	0.26	0.777	1.133
L	1.103	0.197	0.303	0.777	1.608
M	1.103	0.153	0.224	0.777	1.634
N	1.103	0.265	0.316	0.777	1.755
O	1.096	0.167	0.313	0.896	1.574

Working out Internal consistency check:

Table-37 : Theoretical normal deviates(Z_{ij}') derived with width of interval and scale values

Statement	Scale values	Width of the interval					
		0	0.319	0.525	1.103	1.096	1.215
A	1.458	-1.458	-1.139	-0.933	-0.355	-0.362	-0.243
B	1.340	-1.340	-1.021	-0.816	-0.237	-0.244	-0.125
C	1.677	-1.677	-1.358	-1.153	-0.574	-0.581	-0.462
D	1.389	-1.389	-1.070	-0.864	-0.285	-0.292	-0.174
E	1.869	-1.869	-1.550	-1.344	-0.766	-0.773	-0.654
F	1.427	-1.427	-1.108	-0.902	-0.324	-0.330	-0.212
G	1.512	-1.512	-1.193	-0.987	-0.409	-0.416	-0.297
H	1.477	-1.477	-1.158	-0.952	-0.373	-0.380	-0.261
I	1.506	-1.506	-1.187	-0.981	-0.403	-0.410	-0.291
J	1.129	-1.129	-0.810	-0.605	-0.026	-0.033	0.086
K	1.133	-1.133	-0.814	-0.608	-0.030	-0.037	0.082
L	1.608	-1.608	-1.289	-1.084	-0.505	-0.512	-0.393
M	1.634	-1.634	-1.315	-1.109	-0.531	-0.537	-0.419
N	1.755	-1.755	-1.436	-1.230	-0.651	-0.658	-0.540
O	1.574	-1.574	-1.255	-1.049	-0.471	-0.478	-0.359

Table-38 :Theoretical cumulative distribution (P_{ij}') obtained from Z_{ij}'of above table

Statement	1	2	3	4	5	6
A	0.072	0.149	0.175	0.361	0.359	0.404
B	0.09	0.154	0.207	0.406	0.404	0.45
C	0.047	0.087	0.125	0.283	0.281	0.322
D	0.083	0.142	0.194	0.388	0.385	0.431
E	0.048	0.061	0.9	0.222	0.22	0.257
F	0.077	0.134	0.184	0.373	0.372	0.416
G	0.065	0.137	0.162	0.341	0.339	0.383
H	0.07	0.123	0.171	0.355	0.352	0.397
I	0.066	0.138	0.163	0.344	0.341	0.386
J	0.13	0.29	0.272	0.49	0.487	0.466
K	0.013	0.28	0.272	0.488	0.485	0.467
L	0.054	0.099	0.139	0.307	0.304	0.347
M	0.051	0.094	0.134	0.298	0.296	0.338
N	0.04	0.076	0.129	0.258	0.255	0.295
O	0.058	0.105	0.147	0.319	0.317	0.36

Table-39: Difference of theoretical and observed proportions (P_{ij}-P_{ij}')

Statement	1	2	3	4	5	6
A	-0.072	-0.139	-0.165	-0.311	-0.069	0.406
B	-0.08	-0.124	-0.127	-0.246	0.006	0.29
C	-0.037	-0.077	-0.105	-0.203	-0.021	0.308
D	0.437	-0.038	0.025	0.039	0.271	0.444
E	-0.048	-0.061	-0.879	-0.067	0.285	0.527
F	-0.037	-0.084	-0.074	-0.023	0.338	0.534
G	-0.055	-0.097	-0.052	-0.041	0.341	0.507
H	-0.059	-0.101	-0.083	0.008	0.296	0.493
I	-0.066	-0.108	-0.053	0.016	0.289	0.484
J	-0.07	-0.25	0.018	0.07	0.303	0.494
K	-0.003	-0.238	-0.126	0.002	0.265	0.471
L	-0.044	-0.069	-0.048	-0.004	0.302	0.441
M	-0.041	-0.074	-0.052	0.049	0.275	0.57
N	-0.03	-0.035	-0.058	-0.023	0.296	0.542
O	-0.048	-0.075	-0.086	-0.147	0.016	0.286

Absolute discrepancy is worked out by following formula:

$$AD = \frac{.\Sigma\left|pij - pij'\right|}{(noofstatementsXnoofwidths}$$

Here, the obtained value of AD is 16.07/90 =0.18. It seems to be higher than usually obtained values.

Chapter VI

METHOD OF SUMMATED RATINGS

Name of the Experiment: **Development of scale with method of Summated Ratings**

Objective

- To understand the concept of the method of Summated Ratings
- To learn about the methods to compute scale values.

Description

Likert made the weighting of response categories like strongly agree, agree, uncertain, disagree and strongly disagree simpler by assigning a weight of 4 to a strongly agree response for favourable statements and similarly 0 to strongly disagree response to unfavourable statements.

Likert method of scale construction is also called method of summated ratings because each response to a statement may be considered a rating and because those are summated over all statements.

Methodology

- Identification is made of a set of statements that will differentiate between high and low groups. High and low groups are taken as criterion groups to evaluate the individual statements, generally comprising the 25 per cent each of the subjects with the highest and the lowest total scores, respectively.
- Responses are tabulated for high and low group as shown below

The calculation of t for evaluating the difference in the mean response to an attitude statement by a high group and a low group

Response Categories	Low Group				High Group			
	x	f	fx	fx^2	x	f	fx	fx^2
Strongly agree	4	2	8	32	4	15	60	240
Agree	3	3	9	27	3	20	60	180
Uncertain	2	20	40	80	2	10	20	40
Disagree	1	15	15	15	1	4	4	4
Strongly disagree	0	10	0	0	0	1	0	0

- t-value for each statement is calculated with the following formula:

$$t = \frac{\bar{x}_H - \bar{x}_L}{\sqrt{\dfrac{\Sigma(x_H - \bar{x}_H)^2 + \Sigma(x_L - \bar{x}_L)^2}{n(n-1)}}}$$

H= High group

L= Low group

t ≥ 1.75

H&L groups differ significantly

- The statements are arranged in rank order according to their t-values.
- About 20-25 statements with the largest t-value are selected for the attitude scale.

SOLVED EXAMPLE:

(A) TO CALCULATE THE T-VALUE FOR EVALUATING THE DIFFERENCE IN THE MEAN RESPONSE TO IN ATTITUDE STATEMENT BY A HIGH GROUP AND LOW GROUP

Table-40: Response of high and low group for Statement -1

Response Categories	Low Group				High Group			
	X	f	fx	fx^2	X	f	fx	fx^2
SD	4	4	16	64	4	15	60	240
D	3	5	15	45	3	23	69	207
U	2	22	44	88	2	12	24	48
A	1	17	17	17	1	7	7	7
SD	0	12	0	0	0	3	0	0

Table-41: Response of high and low group for Statement -2

Response Categories	Low Group				High Group			
	X	f	fx	fx^2	X	f	fx	fx^2
SD	4	12	48	192	4	18	72	288
D	3	10	3	90	3	22	66	198
U	2	19	38	76	2	10	20	40
A	1	12	12	12	1	8	8	8
SD	0	7	0	0	0	2	0	0

If t ≥ 1.75 the statement have the power to discriminate between the high group and low group of respondents. Since the statement 1 posses a t value equal to 5.56 > 1.75 and statement 2 t value equal to 2.91 > 1.75, both statement are powerful to discriminate between respondents.

(B) Calculation of Weightage of the response categories:

N=200

	Strongly Disagree	Disagree	Uncertain	Agree	Strongly Agree
p	.130	.430	.210	.130	.100
Cp	.130	.560	.770	.900	1.000
Midpoint Cp	.065	.345	.665	.835	.950
Z	-1.514	-.399	.426	.974	1.645
Z+1.514	.000	1.115	1.940	2.488	3.159
Z rounded	0	1	2	2	3

*Midpoint Cp = c. proportions below a given category + 1/2 the proportion within category

(C) Item Analysis:

Response Categories	Low Group				High Group			
	X	**f**	**fx**	**fx^2**	**X**	**f**	**fx**	**fx^2**
SD	4	4	16	64	4	17	68	272
D	3	3	9	27	3	21	63	189
U	2	17	34	68	2	10	20	40
A	1	16	16	16	1	2	2	2
SD	0	10	10	10	0	0	2	0
Sum		50	85	185		50	153	503
		n_L	□x_L	□x_L^2		n_H	□x_H	□x_H^2

The calculated it value is greater then 1.75. Thus, the high and low groups differ significantly, i.e. the statement possesses the power to discriminate between respondents.

ASSIGNMENT

Q1. a. Given below are the frequencies of subjects under respective response categories.

Please work out the respective weightage of the response categories using Likert method of scaling.

Response	Frequency
Strongly Disagree	26
Disagree	86
Uncertain	42
Agree	26
Strongly Agree	20

b. Explain the procedure of item-analysis in method of summated rating. Please comment on the selection of the statement for which the response under low group and high group are as under.

Response Category	Weights	Low group	High Group
Strongly Disagree	4	4	17
Disagree	3	3	21
Uncertain	2	17	10
Agree	1	16	2
Strongly Agree	0	10	0

Chapter-VII
SCALOGRAM ANALYSIS

Name of the Experiment: **Scalogram analysis**

Objective

- To understand the concept of the Scalogram analysis
- To learn about the methods to judge reproducibility of scale items.

Description

Scalogram analysis could be described as a procedure for evaluating sets of statements or existing scales to determine whether or not they meet the requirements of a particular kind of scale, referred as Guttman scale.

A set of statements is said to constitute a unidimensional scale if a person with a more favaourable attitude score than another person must also be as favourable or more favourable in his response to every statement in the set than the other person.

Cornell techniques to evaluate the scalability of the set of statements

- Construction of a table with one column for each response category for each statement and one row for each subject.
- Starting with the person having the highest score, the responses of each subject to each statement are recorded by placing a check mark in the appropriate cell of the table as shown below:

The Cornell techniques applied to a 4 statement scale responded to by 20 subjects. The horizontal lines in the body of the table are possible cutting points for the statements.

Table-42: Response of subjects and their total score

Subjects	Statements								Scores
	1		2		3		4		
	1	0	1	0	1	0	1	0	
1	x		x		x		x		4
2	x			x	x		x		3
3	x		x			x	x		3
4	x		x			x	x		3
5	x			x	x		x		3
6	x			x	x		x		3
7	x			x	x		x		3
8	x			x	x		x		3
9	x			x		x	x		2
10		x	x			x	x		2
11	x			x		x	x		2
12	x			x		x	x		2
13		x	x			x	x		2
14		x		x	x				2

15		x	x			x		x	1
16		x		x		x	x		1
17	x			x		x		x	1
18		x		x	x			x	1
19		x		x		x	x		1
20		x		x		x		x	0
f	12	8	6	14	8	12	16	4	
p and q	.6	.4	.3	.7	.4	.6	.8	.2	
e	1	1	3	1	2	2	2	0	Σ_e=12

- With a perfect scale it is possible to reproduce the responses to the individual statements from knowledge of total scores. However in practical sense, it is not feasible to have a perfect scale. Hence, cutting points for the response categories of each statement are identified.
- A cutting point marks that place in the rank order of subjects where the most common response shifts from on category to the other.
- Guttman's rules for cutting points
- Cutting point should be located so as to minimize error
- No category should have more error in it than non-error
- Errors for the statement are counted
- Sum of the errors for each category for each statement over all statements is obtained
- Co-efficient of reproducibility = $1 - \dfrac{\Sigma e}{total\ response}$

  Σe= sum of errors

- Co-efficient of reproducibility indicate the percent accuracy with which responses to the various statements can be reproduced for the total scores.

Goodenough Method of Scalogram analysis

- A score matrix is prepared with rows corresponding to subjects and column to statements (Table below).
- The responses of a subject are recorded in the row of matrix in terms of the 0 and 1 weights
- The response patterns are recorded with the subject with the highest score assigned to the first row and followed by subsequent next highest score
- Calculate the proportion for responses 1 (p) and proportion for response 0 (q) by dividing the respective sum of scores with total number of subjects for each statement.
- Put the bat chart for each statement such that top part of the bar chart indicates the proportion giving the 1 response to a statement and lower part the proportion of 0 responses (Chart below).
- The point of division is indicated by solid lines and extended through the other bar with dotted lines.
- For four statements scale the possible range of scores will be from 0 to 4.

- Corresponding to each score, a predicted pattern of response to the statements could be determined.
- The predicted pattern of responses for each score is compared with the observed pattern.
- Each deviation of an observed response from predicted response is counted as an error.
- The errors for each subject are summed and recorded

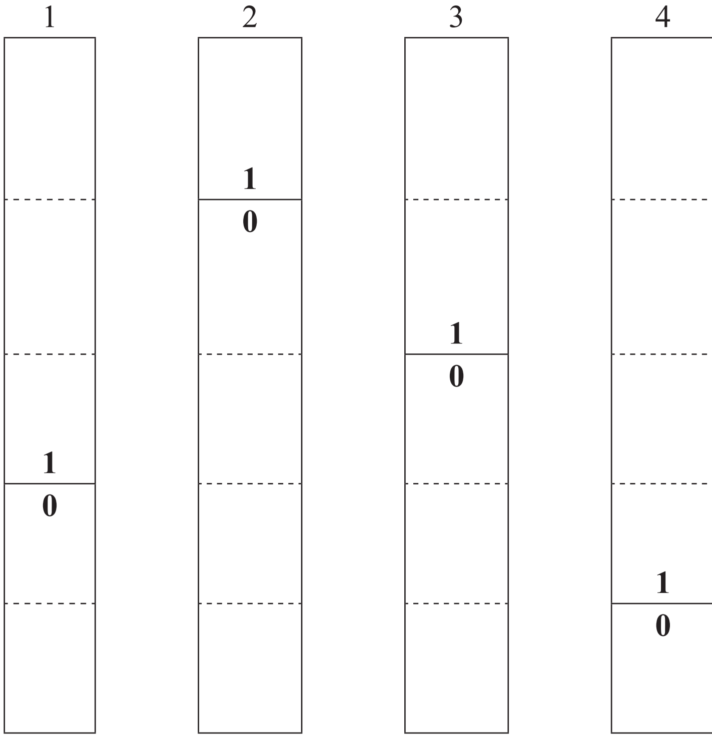
$$\text{Co-efficient of reproducibility} = 1 - \frac{\Sigma e}{\textit{total number of response}}$$

Table-43: A score method for recoding the data of Table

Subjects	Statements				Scores	*e*
	1	2	3	4		
1	1	1	1	1	4	0
2	1	0	1	1	3	0
3	1	1	0	1	3	2
4	1	1	0	1	3	2
5	1	0	1	1	3	0
6	1	0	1	1	3	0
7	1	0	1	1	3	0
8	1	0	1	1	3	0
9	1	0	0	1	2	0
10	0	1	0	1	2	2
11	1	0	0	1	2	0
12	1	0	0	1	2	0
13	0	1	0	1	2	2
14	0	0	1	1	2	2
15	0	1	0	0	1	2
16	0	0	0	1	1	0
17	1	0	0	0	1	2
18	0	0	1	0	1	2
19	0	0	0	1	1	0
20	0	0	0	0	0	0
f	12	6	8	16	42	16
p	**.6**	**.3**	**.4**	**.8**		
q	**.4**	**.7**	**.6**	**.2**		

Bar charts used in determining the predicted response patterns corresponding to the scores of Table

Statements



Score	Predicted Response pattern to statement 1	2	3	4
4	**1**	**1**	**1**	**1**
3	**1**	**0**	**1**	**1**
2	**1**	**0**	**0**	**1**
1	**0**	**0**	**0**	**1**
0	**0**	**0**	**0**	**0**

ASSIGNMENTS

Q1. a) Discuss scalogram analysis. Explain Cornell technique with a suitable example.

b) Determine the coefficient of reproducibility for the responses to statements as mentioned below.

Subjects	1	2	3	4
1	1	1	1	1
2	0	1	1	1
3	1	1	0	1
4	1	1	0	1
5	1	0	1	1
6	1	0	0	1
7	1	1	0	0
8	1	1	0	0
9	0	1	0	1
10	1	0	0	1
11	1	0	1	0
12	1	0	0	0
13	0	1	0	0
14	0	0	0	1
15	0	0	0	0

Chapter-VIII

SCALE DISCRIMINATION TECHNIQUE

Name of the Experiment: **Scale Discrimination Technique**

Objective

- To understand the concept of the scale discrimination technique
- To learn about the methods to compute the scale values and discriminatory power of the individual items.

Description

Edwards and Kilpatrick (1948) devised a method for constructing attitude scales with assumption that a combination of scaling and item analysis procedures would enable one to select a relatively small set of attitude statements from a larger number of available statements such that the set selected would also have a good chance of meeting the requirement of a Guttman scale.

The method of scale construction is called as scale discrimination method because it makes use of Thurston's scaling procedure and retains Likert's procedure for evaluating the discriminatory power of the individual items.

The items selected by this method intend to yield satisfactory co-efficient of reproducibility and to meet the requirements of Guttman's scale analysis. It is essentially a synthesis of the methods of item evaluation of Thurston, Likert and Guttman.

Advantages

- This method eliminates the least discriminating items in a large sample, which Thurston's method alone fails to do.
- Thurston's method by the inclusion of "neutral items", tend to lower reliability and to decrease reproducibility of the set of items finally selected.
- Scale discrimination method offers greater assurance of scalability than any intuitive technique such as applied by Guttman.
- Set of items selected provides a wider range of content than do the intuitive Guttman item.
- In the scale-discrimination method, we obtain items which are not essentially multiple phrasings of the same questions as is often true when the selection of a set of items to be tested for scalability is left to the experience of the investigator.

Methodology

The steps involved are mentioned below:

- Collection of large number of attitude statements relating to the psychological object of interest and their editing based upon informal criteria.
- Obtaining from the judges their degree of favorableness of each statement in terms of 9 or 11 intervals.
- Obtaining scale value and Q value for each statement.

- Plotting the scale and Q values on graph with Q value on Y-axis.
- Rejecting all the statements with Q values above the horizontal line passing through the median Q value. Eliminating the 50 per cent of the statement that show the greatest degree of spread of judgements on the psychological continuum.
- Putting the selected statements in the form of Likert or method of summated rating scale and giving to a new group of about 200 to 300 subjects to indicate their own agreement or disagreement with each of the statements.
- Scoring the responses of subjects with weights of 0 through 5 for the six responses categories, the largest weight given to the response category indicating the most favorable attitude.
- Obtaining total score for each subject based upon his responses to all of the statements.
- Subjecting each statement to item analysis.
- As per Edwards and Kilpatrick method, the top and bottom 27 per cent of subjects referred as high and low groups are selected in terms of total scores on the statements. (as mentioned in table below)
- Dichotomization of response categories referred as high and low groups respectively of statements to combine the response categories by drawing a line between the response so as to minimize the total number of subjects in the low group above the line and the number of subjects in the high group below the line.
- Response categories for all other statements are dichotomized and presented in tables as mentioned below.

Table-44: The distribution of responses to an attitude statement for a low group and a high group

Response Categories	Weights	Low groups *f*	High Group *f*
Strongly agree	**5**	**3**	**38**
Agree	**4**	**5**	**42**
Mildly agree	**3**	**8**	**15**
Mildly disagree	**2**	**26**	**2**
Disagree	**1**	**36**	**2**
Strongly disagree	**0**	**22**	**1**
n		**100**	**100**

Table-45 : Schematic representation for dichotomizing response categories when more than two categories of response are permitted

Response Categories	Low Group	High Group	Total
	a	***b***	***a + b***
	c	***d***	***c + d***
Total	***a + c***	***b + d***	

- Finding the discriminating power of the statements (Phi coefficient)

$$r\Phi = \frac{b \ \ - d}{\sqrt{(a+b)(b+d)(a+c)(c+d)}}$$

- Selection of statements for the scale.
- The Thurston scale values and Phi-coefficient are plotted on horizontal and vertical axis respectively.
- Thurstone continuum divided into half scale intervals and from each half scale interval statements with the highest Phi-coefficients are selected.
- Testing of reproducibility with calculation of coefficient of reproducibility by dividing the selected statements in two forms of scale and applying scalogram analysis separately for both the sets of statements.

ASSIGNMENT

Q1. Describe scale discrimination technique. Please comment on discriminating power of two statement for which response patter under low group and high group are as under.

Response Category	**Weights**	**Statement 1**		**Statement 2**	
		Low group	**High group**	**Low group**	**High group**
Strongly Disagree	5	3	38	2	41
Agree	4	5	42	6	35
Mildly agree	3	8	15	9	20
Mildly disagree	2	26	2	35	2
Disagree	1	36	2	27	1
Strongly Agree	0	22	1	21	1

Chapter-IX

RELIABILITY & VALIDITY

Name of the Experiment: **Computation of Reliability and Validity**

Objective

- To understand the concept of the Reliability and Validity.
- To learn about the methods to estimate and judge Reliability and Validity of instruments.

Description

Reliability is the accuracy or precision of a measuring instrument.

Reliability is the proportion of the true variance to the total obtained variance of the data yielded by a measuring instrument.

$$r_{tt} = \frac{v_e}{v_t}$$

Reliability is the proportion of error variance to the total obtained variance yielded by a measuring instrument subtracted from 1.00

$$r_{tt =}\ 1 - \frac{v_e}{v_t}$$

$$\text{or}\ \ r_{tt =} \frac{v_t - v_e}{v_t}$$

Approaches to the estimation of reliability

There have been three standard procedures known as split-half, alternate forms and test-retest methods. All have in common the goal of deriving two sets of scores form the “same” test administered to the “same” sample for the purpose of correlation to find r_{tt}.

In case of the split half method, the Spearman-Brown formula has usually been applied to estimate the reliability of the test of full length from the obtained estimate of correlation of a test of half length.

The division of a test into two parts should be so accomplished that each should represent faithfully the total test in all significant aspects. Coefficient of reliability concerns the equivalence of parts for measurement purpose i.e. internal consistency.

A retest coefficient of correlation tells nothing about the internal consistency rather answers the question concerning how stable or dependable are the measurements over a period of time.

The analysis of variance approach to reliability

According to Hoyt. the matrix of item scores is regarded as a two-way factorial design for analysis of variance with replications.

Hoyt's basic formula for reliability is:

$$r_{tt} = 1 - \frac{v_e}{v_{ind}}$$

$$= \frac{v_{ind} - v_e}{v_e}$$

V_{ind} = Variance for the individuals

V_e = Variance for the error

VALIDITY

A test is said to be valid if it measures what it intends to measure. The degree to which a test measures what it measures may be called its intrinsic validity. The degree to which a test measures factors that are common to other measures may be called as relevant validity. The index of relevant validity is the square root of the test's communality (h^2).

Three types of validity viz. content, criterion- related and construct are commonly referred. Content validity is the representativeness or sampling adequacy of the content of a measuring instrument. It guides whether the content of the measuring instrument is representative of the content or universe of content of the property being measured. In criterion related validity the emphasis is on the criterion and its prediction. Construct validity seeks to explain individual difference in the test scores of a measuring instrument. It delineates what proportion of the total test variance is accounted for by the construct.

ASSIGNMENTS

Give below in the table are the item scores and total scores of 10 individuals in a 12 items test. Estimate the reliability by the approaches of:

(a) Split-half method

Spearman Brown prophecy formula

$$r_{oe} = \frac{r_{\phi}\,\sigma_t - \sigma_0}{\sigma_t^{\,2} + \sigma_0^{\,2} - 2r_{\phi}\,\sigma_0\sigma_t}$$

(b) Kuder-Richardson formula

$$r_{tt} = \left(\frac{n}{n-1}\right)\left(\frac{\sigma^2{}_t - \Sigma pq}{\sigma^2{}_t}\right)$$

n = number of items in the test

p = proportion of correct responses to each item

q = 1 – p

Table-46: Score pattern over 12 scale items of 10 respondents

		Items											
		a	***b***	***c***	***d***	***e***	***f***	***g***	***h***	***i***	***j***	***k***	***l***
Persons	1	1	0	1	0	0	0	0	0	0	0	0	0
	2	1	1	1	0	0	1	0	0	0	0	0	0
	3	1	1	1	1	0	0	0	0	0	0	0	0
	4	1	1	0	1	1	0	0	1	0	0	0	0
	5	1	1	1	1	1	0	0	0	0	0	0	0
	6	1	1	1	0	1	1	1	0	0	0	0	0
	7	1	1	1	1	1	1	1	0	0	0	0	0
	8	1	1	1	1	0	1	1	1	1	1	0	0
	9	1	1	1	1	1	1	1	1	1	1	1	0
	10	1	1	1	1	1	1	1	1	1	1	1	1
	R	10	9	9	7	6	6	5	4	3	3	2	1
	W	0	1	1	3	4	4	5	6	7	7	8	9
	P	1.0	.9	.9	.7	.6	.6	.5	.4	.3	.3	.2	.1
	P	1.0	.81	.81	.49	.36	.36	.25	.16	.09	.09	.04	.01
	P	.0	.09	.09	.21	.24	.24	.25	.24	.21	.21	.16	.09

Assignments 2 : Using ANOVA approach compute the reliability coefficient of the instrument containing six-point scale to measure attitude towards Pusa Basmati 1121 variety of Paddy. The scores of five farmers with respect to four test items are given in the table below:

	Items			
Individuals	**A**	**B**	**C**	**D**
1	6	6	5	4
2	4	6	5	3
3	4	4	4	2
4	3	1	4	2
5	1	2	1	1

Step

- Calculate the correction factor $c = \frac{(\Sigma x)^2}{N}$
- Calculate the total sum of squares
  - Between the items
  - Between the individuals
- Calculate the Degree of freedom
- Put the data in ANOVA Table as below

Source of variance	Degree of (df) freedom	Sum of squares (SS)	Mean sum of squares (MSS)	F Ratio
Items	(n-1)			
Individuals	(r-1)			
Residual	(n-1) (r-1)			